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3 (Sem-3/CBCS) PHY HC 1

2024

PHYSICS

(Honours Core)

Paper : PHY-HC-3016

(Mathematical Physics-II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer the following questions : $1 \times 7 = 7$

(a) Define regular singular point at $x = 0$ for a second order differential equation.

(b) The function $P_n(1)$ is given as

(i) -1

(ii) zero

(iii) 1

(iv) $P_n(-1)$

(Choose the correct option)

Contd.

(ii) By using the Cayley-Hamilton theorem, compute the inverse of

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

(d) What is adjoint of a matrix? For the

matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ verify the theorem

$A \cdot (\text{Adj } A) = (\text{Adj } A) \cdot A = |A| I$, where I is unit matrix. 1+4=5

(e) Find the Fourier series representing $f(x) = x, 0 < x < 2\pi$.

4. (a) (i) Find the eigenvalue and eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}.$$

(ii) Show that any arbitrary square matrix can be represented as a sum of symmetric and skew-symmetric matrix. 4

OR

- (b) (i) Prove the Rodrigues' formula for Legendre's polynomials

$$P_n(x) = \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n} (x^n - 1)^n. \quad 6$$

- (ii) Show that

$$H_0(x) = 1 \text{ and } H_1(x) = 2x.$$

2+2=4

- (a) (i) Solve the following equation by the method of separation of variables

$$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y},$$

$$\text{given } u(0, y) = 8e^{-3y} \quad 5$$

- (ii) Write the recursion formula of gamma function. Prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}. \quad 1+4=5$$

OR

- (b) (i) A square wave function is represented as

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x < 0 \\ L, & \text{for } 0 \leq x < \pi \end{cases}$$

Draw the graphical representation of the given function. Show that the Fourier expansion of the function is given as

$$f(x) = \frac{L}{2} + \frac{2L}{\pi} \sum_{n=1}^{\infty} \frac{\sin nx}{n} \quad (\text{for } n \text{ odd})$$

1+6=7

- (ii) Write the orthogonality conditions of sine and cosine functions. 3

- (a) Prove the following recurrence relations: 3+2+5=10

(i) $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$

(ii) $H_n^1(x) = 2xH_n(x) - H_{n+1}(x)$

(iii) $xP_n^1(x) - P_{n-1}^1(x) = nP_n(x)$

OR

(b) (i) Show that the matrix

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \text{ is unitary.}$$

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(ii) Find the power series solution of the differential equation

$$\frac{d^2y}{dx^2} + w^2y = 0 \text{ in powers of } x. \quad 7$$